Exercise 33

Find the limit or show that it does not exist.

$$\lim_{x\to-\infty}(x^2+2x^7)$$

Solution

Make the substitution, $u=-x$, so that as $x\to-\infty$, $u\to\infty$. Then factor the highest power of u in order to turn this sum into a product with an inverse power of u.

$$\begin{aligned}
\lim_{x\to-\infty}(x^2+2x^7) &= \lim_{u\to\infty}[(-u)^2+2(-u)^7] \\
&= \lim_{u\to\infty}[u^2+2(-u^7)] \\
&= \lim_{u\to\infty}(u^2-2u^7) \\
&= \lim_{u\to\infty}\left[u^7\left(\frac{1}{u^5}-2\right)\right] \\
&= \left(\lim_{u\to\infty}u^7\right)\left[\lim_{u\to\infty}\left(\frac{1}{u^5}-2\right)\right] \\
&= \left(\lim_{u\to\infty}u^7\right)\left(\lim_{u\to\infty}\frac{1}{u^5}-\lim_{u\to\infty}2\right) \\
&= \left(\lim_{u\to\infty}u^7\right)(0-2) \\
&= (\infty)(-2) \\
&= -\infty
\end{aligned}$$